Sets

23-10-2018

Set: Collection of distinct objects

↓

(Not Repeated)

Set Notations:

1- Tabular form:

$A = \{1, 2, 3, 4, 5\}$

② Descriptive form (described as a Sentence)

A = set of first five positive Integers

③ Set build form

set is decribed by a Rule.

$$A = \{x \mid x \in Z^+ \ \& \ x \leq 5\}$$

$$A = \{x \mid 1 \leq x \leq 5\}$$

Write elements of the set.

① $B = \{x : x^2 - 4 = 0\}$

$x^2 - 4 = 0$ $\quad x = 2$ $\quad x = -2$

$x^2 = 4$ $\quad \{-2, 2\}$

$x = \pm\sqrt{4}$

(ii) $C = \{x : x \text{ is a perfect square } \& \ x < 20\}$

$C = \{1, 4, 9, 16\}$

(iii) Absolute value $|x|$

$D = \{x : |x| = 4\}$

$D = \{-4, 4\}$

Set Language

(1) Element Symbol : $\in$

$\in$: is an element of
: belongs to
: is present in
: lies in
: is inside.

e.g:

$$A = \{1, 2, 3, ---, 10\}$$

$2 \in A \longrightarrow$ True

$7 \in A \longrightarrow$ True

$13 \in A \longrightarrow$ false

$\{4\} \in A \longrightarrow$ false.

Set | not element

(2) $\notin$ or $\in'$

: Not an element of
: does Not belong to
: is Not present in
: does Not lie in
: is Not in

$4 \notin A \Longrightarrow$ false.

$0 \notin A \longrightarrow$ True.

(3) Number of elements

$n(A)$ → Number of elements in set A

$A = \{1, 2, 3, ---, 10\}$

$n(A) = 10$

(4) Empty Set : ϕ

⇒ Called NULL Set

it does Not contain any element.

$B = \{\ \}$

$B = \phi$

$n(B) = 0$

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(5) Universal set $\mathcal{E}$ ($\mathcal{E}$)

A Set that contains all the elements of given sets.

$n(\mathcal{E}) \geq$ than given Number of elements.

Relationships between ⑤ Two sets:

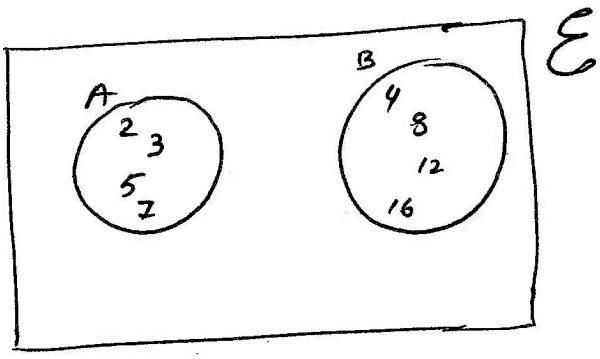
① Disjoint Sets

The sets have No common elements.

$A = \{2, 3, 5, 7\}$ prime:

$B = \{4, 8, 12, 16\}$ Multiples of: 4

They don't have any common element.



venn diagram

② Overlapping Sets:

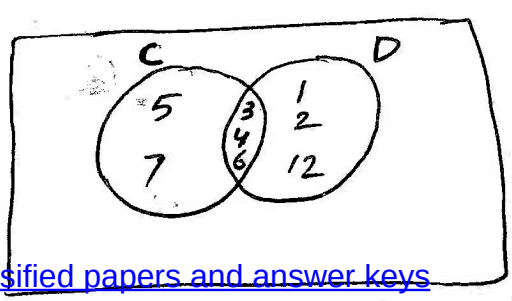
The sets have Some common elements & Some uncommon elements.

$C = \{3, 4, 5, 6, 7\}$

$D = \{1, 2, 3, 4, 6, 12\}$ ← factors of 12

Common: 3, 4, 6

Uncommon: 1, 2, 5, 7, 12



(3) Subset $\subset$ $\subseteq$ (Proper Subset)

A set P is a subset of set Q if all the elements of P are also present in Q.

$P \subset Q \Rightarrow$ P is subset of Q

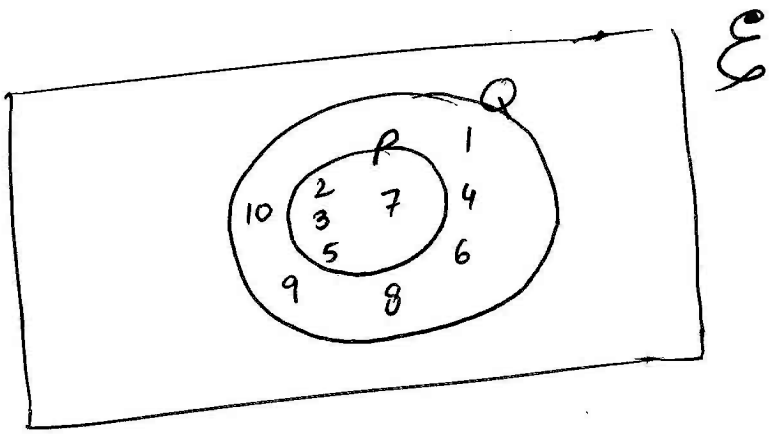
Q is Superset of P.

Q contains set P inside it.

$P = \{2, 3, 5, 7\}$ first four prime Numbers.

$Q = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

All elements of P $\{2, 3, 5, 7\}$ are inside Q.



$n(P) = 4$

$n(Q) = 10$

Making Subset:

if a Set 'A' has n elements

then **Number of Subsets = 2^n**

e.g $A = \{1, 2, 3\}$

How many Subsets = 2^3

= 8 Subsets

List:

$\{\}, \{1\}, \{2\}, \{3\}$

$\{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$

List all subsets of $\{a, b, c, d\}$

$\{\}, \{a\}, \{b\}, \{c\}, \{d\}$

$\{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{b,d\}$

$\{c,d\}, \{a,b,c\}, \{a,c,d\}, \{b,c,d\}$

$\{a,b,d\}, \{a,b,c,d\}$

$2^4 = 16$ Subsets

Operations in Sets:

(1) Compliment A' or A^c

A' ⇒ all the element of $\mathcal{E}$ except A.

$$A' = \mathcal{E} - A$$

A' : outside A

A' : other than A

A' : Not in A

A' : everything except A.

ex:

$$X = \{2, 3, 5, 7\}$$

$$\mathcal{E} = \{1, 2, 3, ---, 10\}$$

$$X' = \mathcal{E} - X$$

$$= \{1, 4, 6, 8, 9, 10\}$$

$$n(X') = 6$$

$$\mathcal{E}' = \phi$$

$$= \mathcal{E} - \mathcal{E}$$

$$= \{\}$$

(2) Union (A ∪ B)

Union of Two Sets A and B contains all the elements of A as well as B.

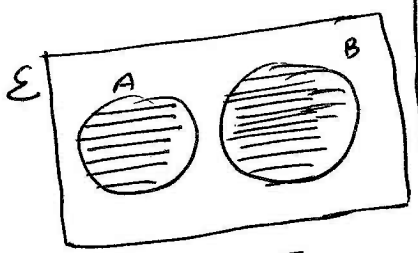
(Do Not repeat the common)

A ∪ B → everything of A & B

→ All A as well as all B

$A = \{1, 2, 3, 4, 5\}$	$n(A) = 5$
$B = \{3, 4, 5, 6, 7, 8\}$	$n(B) = 6$
$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$	$n(A \cup B) = 8$

Venn diagrams → examples

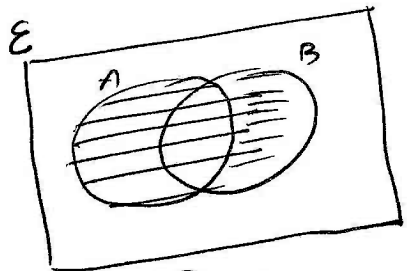


$A \cup B$: ≡

if A & B are disjoint

$A \cup B = A + B$

$n(A \cup B) = n(A) + n(B)$



$A \cup B$: ≡

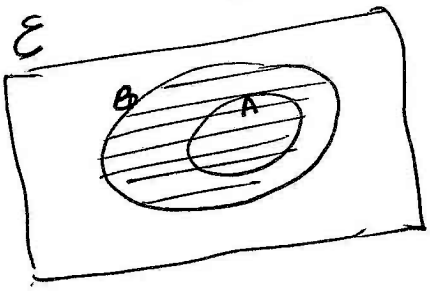
if A & B are overlapping :

$n(A \cup B) < n(A) + n(A)$

less than

Subset

$A \subset B$



$A \cup B$

if $A \subset B$ then $A \cup B = B$

$n(A \cup B) = n(B)$

(3) Intersection : $A \cap B$

→ Intersection contains only the common elements.

$A \cap B$: A intersection B

$A \cap B$: Common of A & B

$A \cap B$: part of A that is in B.

$A \cap B$: Inside A but also inside B

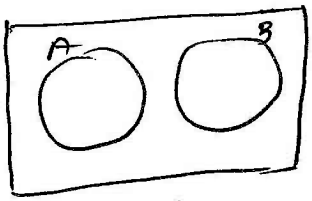
$A \cap B$: only the overlapped region.

$A = \{1, 2, 3, 4, 5\}$

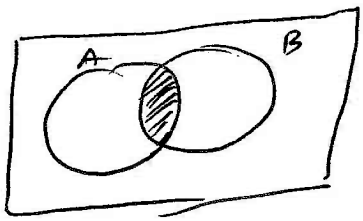
$B = \{3, 4, 5, 6, 7, 8\}$

$A \cap B = \{3, 4, 5\}$

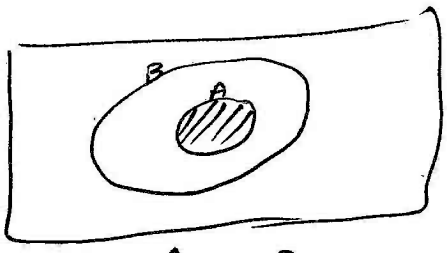
Venn diagram examples



$A \cap B = \emptyset$



$A \subset B$



if $A \subset B$
then $A \cap B = A$

Combined Complement

$$(A \cup B \cup C)' = A' \cap B' \cap C'$$

$\cup \longrightarrow \cap$ $\cap \longrightarrow \cup$	$(A')' = A$

$$(A \cap B)' = A' \cup B'$$

$$(A \cup B' \cap C)' = A' \cap B \cup C'$$

$$(A \cup B')' = A' \cap B$$

⑤ Subtraction of Two Sets :

$A - B$ or A/B

All the elements from set (A) that are Not in B.

$A = \{1, 2, 3\}$ $\quad B = \{2, 3, 4, 5, 6, 7, 8\}$

$A - B = \{1\}$

Some Common Rules:

$$(A')' = A$$

$$\mathcal{E}' = \phi$$

$$A \cap A = A$$

$$A \cup A = A$$

$$A \cap \mathcal{E} = A$$

$$A \cup \mathcal{E} = \mathcal{E}$$

$$A \cup A' = \mathcal{E}$$

$$A \cap A' = \phi$$

if $A \subset B$

$A \cup B = B$

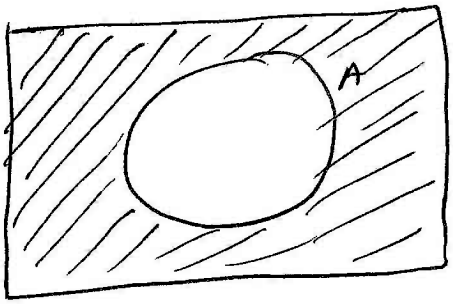
$A \cap B = A$

distributive properties:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

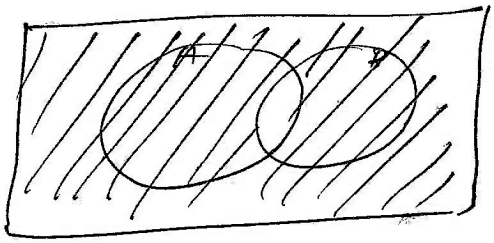
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Shading in Venn diagrams

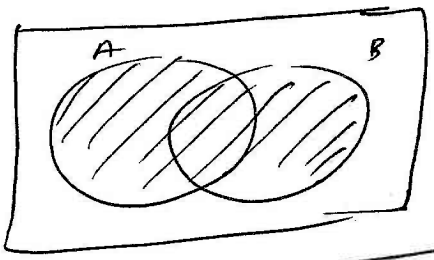


A'

$(A \cap B)' = A' \cup B'$

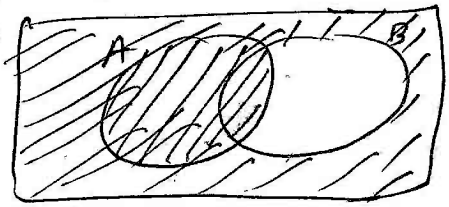


$A \cup B$

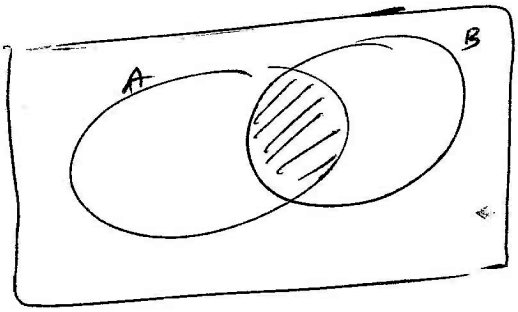


$A \cup B'$

also

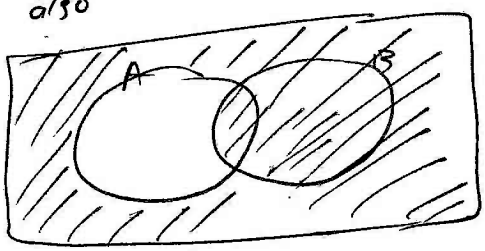


$A \cap B$



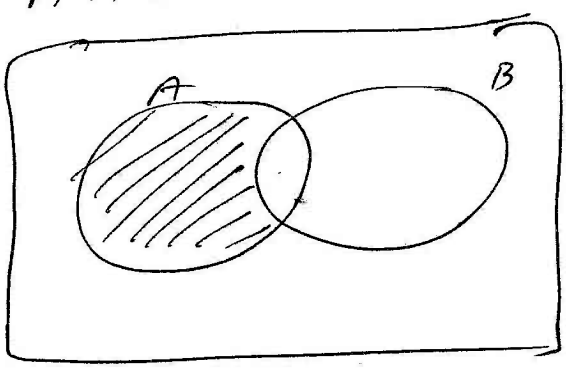
$A' \cup B$

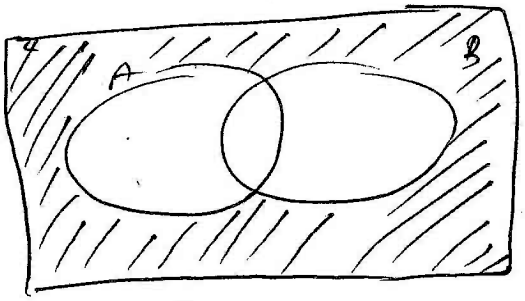
also



Bad

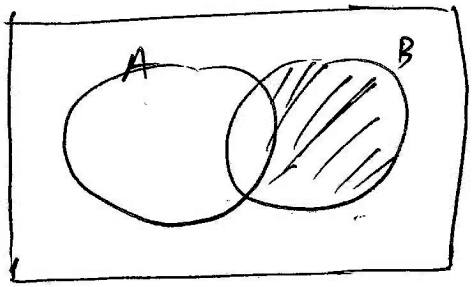
$A \cap B'$



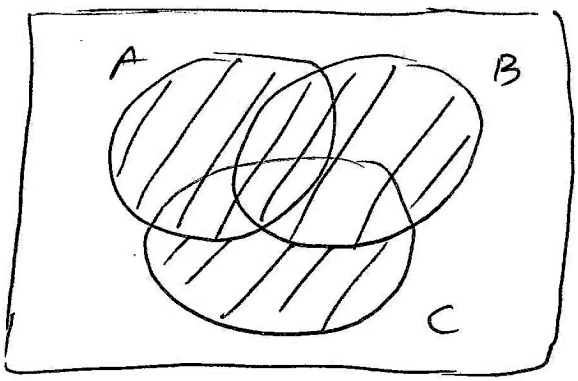


$(A \cup B)'$ $A' \cap B'$

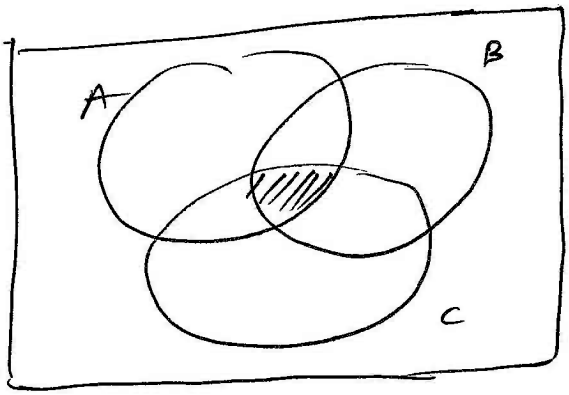
$A' \cap B$

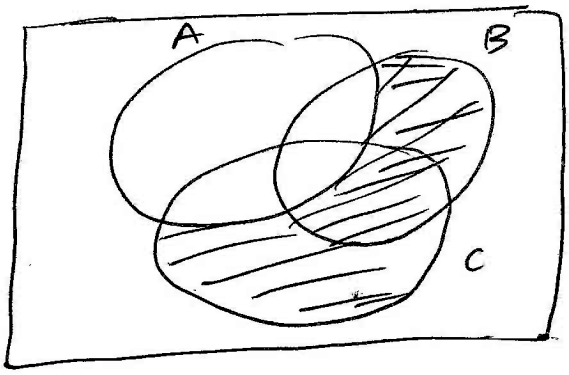


$A \cup B \cup C$

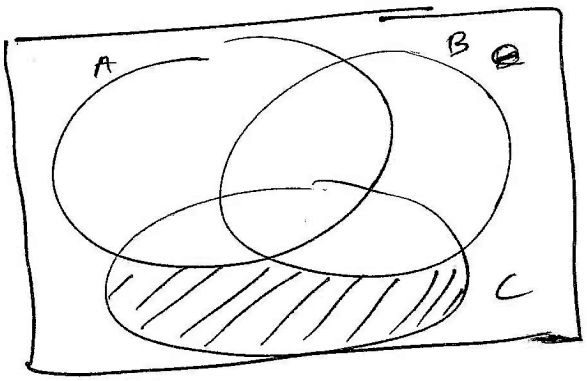


$A \cap B \cap C$

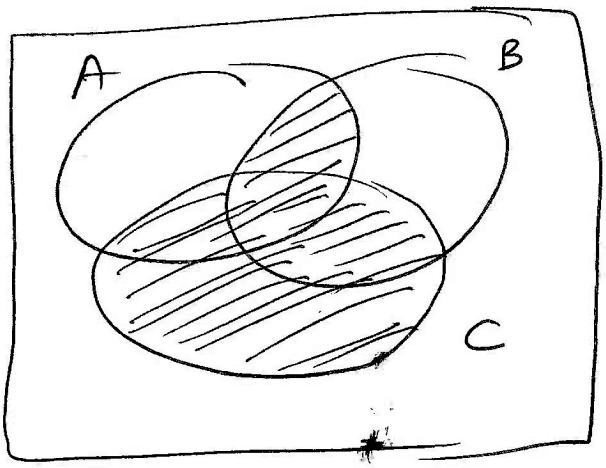




$A' \cap (B \cup C)$

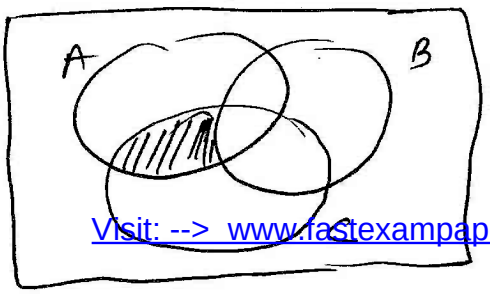


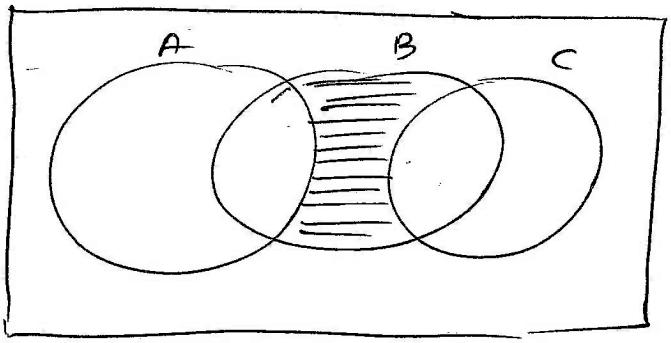
$C \cap (A \cup B)'$



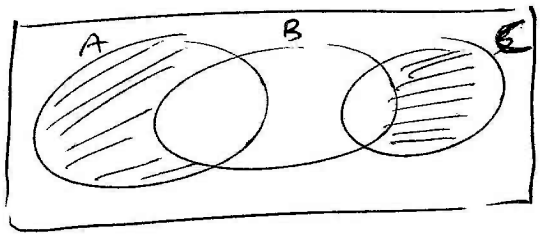
$C \cup (A \cap B)$

$(A \cap C) \cap B'$

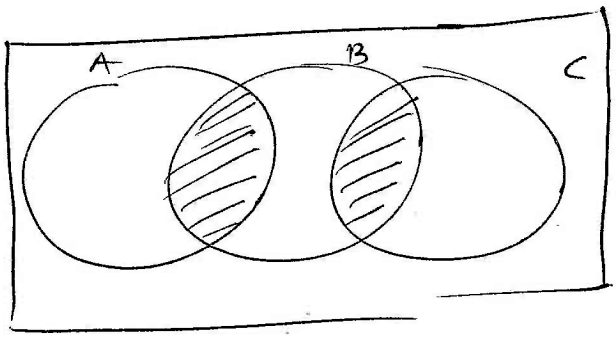




$B \cap (A \cup C)'$



$(A \cup C) \cap B'$



$B \cap (A \cup C)$

Mr. Afdal

23/10/2018

Coming Next: logical problems Solving using Sets & Venn diagrams.